1.) $2x^2 coty \frac{dy}{dx} = 5x - 3siny$ diferansiyel denklemini çözünüz. (% 50)

2.) $\frac{3x^2y+1}{y}dx - \frac{x}{y^2}dy = 0$ diferansiyel denklemini çözünüz. (%50)

C.1)

$$\left.\begin{array}{l} \sin y = u \\ \cos y\, y' = u' \end{array}\right\} \quad 2x^2\frac{u'}{u} = 5x - 3u$$

$$\frac{2x^2u'}{2x^2} = \frac{(5x-3u)u}{2x^2}$$

$$u' = \frac{(5x-3u)u}{2x^2}$$

Denklemi daha açık biçimde yazarsak,

$$u' - \left(\frac{5x}{2x^2}\right)u = -\frac{3u^2}{2x^2} \qquad \left[\, y' + P(x)y = G(x)y^n = \text{Bernoulli dif. denk.} \right]$$

$P(x) = -\frac{5x}{2x^2} = -\frac{5}{2x}$; $n=2$

$G(x) = -\frac{3}{2x^2}$;

$u^{1-n} = t$ dönüşümü yaparsak,

$$t' + (1-2)\left(-\frac{5}{2x}\right) = (1-2)\frac{-3}{2x^2}$$

$$t' + \frac{5}{2x} = +\frac{3}{2x^2} \quad \text{(Lineer tip dif. denk.)}$$

$$t = u^{1-2} = (\sin y)^{1-2} = \left[\int +\frac{3}{2x^2} e^{\int\frac{5}{2x}dx} dx + c\right] e^{\int -\frac{5}{2x}dx} \quad ; \quad e^{\int\frac{5}{2x}dx} = e^{\frac{5}{2}\ln x} = x^{\frac{5}{2}}$$

$$e^{\int -\frac{5}{2x}dx} = e^{-\frac{5}{2}\ln x} = \frac{1}{x^{5/2}}$$

$$\frac{1}{\sin y} = \left[\int +\frac{3}{2x^2}x^{\frac{5}{2}}dx + c\right]\frac{1}{x^{5/2}} = \left[+\frac{3}{2}\int x^{\frac{1}{2}}dx + c\right]\frac{1}{x^{5/2}}$$

$$\frac{1}{\sin y}=\left[+\frac{3}{2}\,\frac{x^{3/2}}{3/2}+c\right]\frac{1}{x^{5/2}}=\left[+\frac{3}{2}\cdot\frac{2}{3}x^{3/2}+c\right]\frac{1}{x^{5/2}}$$

$$\frac{1}{\sin y}=\left(x^{3/2}+c\right)\frac{1}{x^{5/2}}$$ bulunur.

C.2) $\frac{3x^2y+1}{y}dx-\frac{x}{y^2}dy=0$; $P(x,y)=3x^2+\frac{1}{y}$

$Q(x,y)=-x/y^2$

$P'_y \overset{?}{=} Q'_x$

$0-\frac{1}{y^2}=-\frac{1}{y^2}$ olduğundan tam diferansiyel denklemdir.

Tam dif. denklemlerin genel çözümü;

$$\int_a^x P(x,y)dx+\int Q(a,y)dy=c$$

$$\int_a^x\left(3x^2+\frac{1}{y}\right)dx+\int-\frac{a}{y^2}dy=c \longrightarrow \left[3\frac{x^3}{3}+\frac{x}{y}\right]_a^x+a\cdot\frac{1}{y}=c$$

$$x^3+\frac{x}{y}-a^3-\cancel{\frac{a}{y}}+\cancel{\frac{a}{y}}=c$$

$$\boxed{x^3+\frac{x}{y}=c}$$